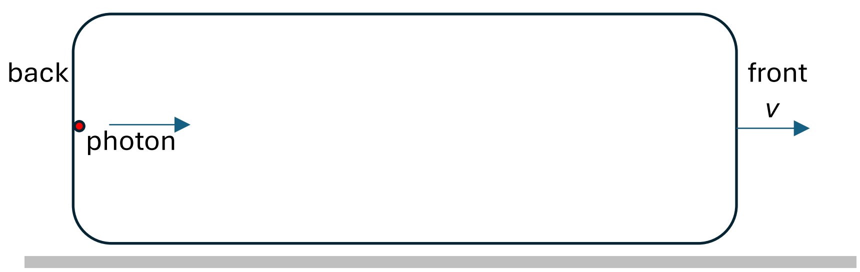
Teacher notes
Topic A

An instructive relativity problem

A rocket of proper length *L* moves with speed *v* relative to the ground. A photon is emitted from the back of the rocket and is received at the front. The gamma factor for speed *v* is γ.



What is the time taken according to an observer on the ground?

This can be answered very quickly simply by applying a Lorentz transformation: we have that

$$\Delta t = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)$$

with $\Delta t' = \frac{L}{c}$ and $\Delta x' = L$. This gives the answer

$$\Delta t = \gamma\left(\frac{L}{c} + \frac{v}{c^2}L\right) = \frac{\gamma L}{c}\left(1 + \frac{v}{c}\right)$$

But it is instructive to also understand this with a more intuitive approach.

As far as the ground observer is concerned, the photon will have to move the length of the rocket (which is the Lorentz contracted length $\frac{L}{\gamma}$) but in the meantime the front of the rocket will move forward. If the trip takes time *T* for the ground observer, the front of the rocket will move a distance *vT*. Hence the distance travelled by the photon is $\frac{L}{\gamma} + vT$. The speed of the photon is *c* so $\frac{L}{\gamma} + vT = cT$. This gives

$$T(c-v)=\frac{L}{\gamma}$$

$$T\left(1-\frac{v}{c}\right)=\frac{L}{\gamma c}$$

$$T=\frac{\frac{L}{\gamma c}}{1-\frac{v}{c}}$$

This looks hopelessly different from the answer we got through the Lorentz transformation. However,

$$T=\frac{\frac{L}{\gamma c}}{1-\frac{v}{c}}=\frac{\frac{L}{\gamma c}}{1-\frac{v}{c}}\frac{1+\frac{v}{c}}{1+\frac{v}{c}}=\frac{L}{\gamma c}\frac{1+\frac{v}{c}}{1-\frac{v^2}{c^2}}=\frac{L}{\gamma c}\frac{1+\frac{v}{c}}{\frac{1}{\gamma^2}}=\frac{\gamma L}{c}\left(1+\frac{v}{c}\right)$$

and is identical to the previous answer! It always pays to get an answer in two different ways.

We can continue this by asking the time taken for the photon to be reflected back to the rear of the rocket. Obviously, the answer is $\Delta t=\frac{\gamma L}{c}\left(1-\frac{v}{c}\right)$ simply by reversing the sign of *v*. The return trip is shorter since the back of the rocket is now approaching the photon. The total time (back to front to back) is then

$\frac{\gamma L}{c}\left(1+\frac{v}{c}\right)+\frac{\gamma L}{c}\left(1-\frac{v}{c}\right)=\frac{2\gamma L}{c}$ for the ground observer. For the rocket observer it is just $\frac{2L}{c}$ and this is a proper time interval for the rocket observer. The ground observer will then measure the dilated time interval of $\gamma\times\frac{2L}{c}$ precisely as we found!